

**#0 Theta Bowl**  
**MAΘ National Convention 2018**

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A car driving 30 miles per hour is equivalently driving  $A$  feet per second.

Let  $B$  be the number of distinct permutations of the letters in the word REVOLVER.

Find the remainder when  $B$  is divided by  $A$ .

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A triangle has sides with lengths 3, 3, and 4 inches. Let  $A$  be the number of square inches in the area of the region bounded by the triangle.

An isosceles trapezoid has sides with lengths 6, 6, 6, and 14 inches. Let  $B$  be the number of square inches in the area of the region bounded by the trapezoid.

Find the value of  $\frac{A \cdot B}{(A+B)^2}$ .

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If 9 lawyers can process 6 files in 14 hours,  $A$  lawyers can process 4 files in 7 hours. Assume the lawyers work cooperatively and independently.

$B$  is a seven-digit palindrome in which the second digit is double the first digit, the fifth digit is twice the fourth digit, and the sum of all the digits is 26.

Find the remainder when  $B$  is divided by  $A$ .

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**#3 Theta Bowl**  
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The sixth grade at Mu Alpha Theta Middle School consists of 190 right-handed students and 24 left-handed students. Among these students, there are seven times as many right-handed females as left-handed females, and there are nine times as many right-handed males as left-handed males. What is the probability, written as a decimal rounded to the nearest thousandth, that a right-handed student selected randomly from the sixth grade at Mu Alpha Theta Middle School is female? Assume each sixth grade student at Mu Alpha Theta Middle School is 1) either male or female, and 2) either left- or right-handed.

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$$A = \frac{\sum_{n=1}^{16} n^2}{\sum_{n=1}^{33} n^2}$$

$$B = \begin{vmatrix} 1 & 5 & 8 \\ -3 & 0 & 2 \\ 6 & -4 & -2 \end{vmatrix}$$

Find the product  $A \cdot B$ .

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A right triangle has one leg with length 2. If the product of the lengths of the median, angle bisector, and altitude to the hypotenuse of this triangle is  $9\sqrt{2}$ , find the area enclosed by the circle that circumscribes this triangle.

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**#6 Theta Bowl**  
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Ten club members are to be seated at a round table. Five of the members are officers and may sit in any order at the table, as long as they are all seated in adjacent chairs. The other five members may sit in any order at the table in the remaining chairs. In how many distinguishable ways may the people in this club be seated? Two ways are distinguishable if one is not a rotation of the other.

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**#7 Theta Bowl**  
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Two positive real numbers  $x$  and  $y$ , where  $x > y$ , are such that  $\sqrt{x} + \sqrt{y} = 10$ . Let  $A$  and  $H$  be the arithmetic and harmonic means, respectively, of  $x$  and  $y$ . If the geometric mean of  $A$  and  $H$  is 8, find the numerical value of  $x$ .

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**#8 Theta Bowl**  
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Solve the inequality  $\frac{x}{x+2} + \frac{1}{x-1} \leq \frac{3}{2}$ . Write your answer in interval notation.

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The equation of the ellipse with center at  $(2, -3)$ , one focus at  $(10, -3)$ , and an enclosed area of  $255\pi$  can be written in the form  $225x^2 + By^2 + Cx + 1734y - E = 0$  for real numbers  $B$ ,  $C$ , and  $E$ . Find the greatest positive prime integral divisor of  $E$ .

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In abstract algebra, we use the term “symmetries” of a polygon to refer to both reflection of the polygon about a line or rotation of the polygon by some number of degrees  $\theta$ ,  $0^\circ \leq \theta < 360^\circ$ , so that the polygon is in the exact same position (basically mapping one vertex to another, potentially to the same vertex). For example, for a square, there are 8 symmetries: rotation about its center by  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , or  $270^\circ$ ; reflection across either diagonal, or reflection about the lines that connect the midpoints of alternate sides of the square. For an isosceles triangle, there are 2 symmetries: rotation about its center by  $0^\circ$  or reflection about its altitude to the unequal side.

Let  $A$  be the number of symmetries of a regular 2018-gon,  $B$  be the number of symmetries of a rectangle with one dimension twice as long as the other, and  $C$  be the number of symmetries of a scalene triangle. Find the product  $A \cdot B \cdot C$ .

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Find the sum of all real solutions to the equation  $\log(1-3x) = \log_{100}(7x^2 - 2x + 17)$ .

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Find the sum of all real solutions to the equation  $\log(1-3x) = \log_{100}(7x^2 - 2x + 17)$ .

**#12 Theta Bowl**  
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Consider a rectangular puzzle, in which any row has the same number of pieces as any other row, and any column has the same number of pieces as any other column (this is the layout of a traditional rectangular puzzle). Let the dimensions of such a puzzle be the number of pieces in a row by the number of pieces in a column.

There are two rectangular puzzles whose number of border pieces equals the number of interior pieces. Let their dimensions be  $A \times B$  and  $C \times D$ .

There is one rectangular puzzle whose number of border pieces is exactly twice that of its interior pieces. Let its dimensions be  $E \times F$ .

There are four rectangular puzzles whose number of interior pieces is exactly twice that of its border pieces. Let their dimensions be  $G \times H$ ,  $I \times J$ ,  $K \times L$ , and  $M \times N$ .

Find the numerical value of  $A+B+C+D+E+F+G+H+I+J+K+L+M+N$ .

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There are four rectangular puzzles whose number of interior pieces is exactly twice that of its border pieces. Let their dimensions be  $G \times H$ ,  $I \times J$ ,  $K \times L$ , and  $M \times N$ .

Find the numerical value of  $A+B+C+D+E+F+G+H+I+J+K+L+M+N$ .

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For the polynomial  $g(x) = 3x^5 - 4x^4 + 7x^3 - 12x - 8$ , find the value of  $g(1) + g(2) + g(3) + g(4)$ .

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For the polynomial  $g(x) = 3x^5 - 4x^4 + 7x^3 - 12x - 8$ , find the value of  $g(1) + g(2) + g(3) + g(4)$ .

**#14 Theta Bowl**  
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Write in  $a + bi$  form, where  $a$  and  $b$  are real numbers and  $i = \sqrt{-1}$  :  $\frac{535 + 283i}{23 + 37i}$

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